## 2023 Algebra II Exam

- 1. What is the domain of the function  $f(x) = \sqrt{1 \sqrt{x}}$ ? (a)  $(0, \infty)$  (b)  $[0, \infty)$  (c) (0, 1] (d) [0, 1] (e) [1, 0)
- 2. What is the y-intercept of the line passing through A(-2,4) and B(1,10)?
  - (a) 10 (b) 4 (c) 5 (d) 2 (e) 8
- 3. Find the equation of the line graphed below.



4. Find the coordinates of the point where the following two lines intersect:

$$2x + y = 10$$

$$4x - y = 5$$
(a)  $\left(\frac{5}{2}, 5\right)$ 
(b)  $\left(\frac{7}{10}, -7\right)$ 
(c)  $\left(7, \frac{7}{4}\right)$ 
(d)  $\left(-5, \frac{13}{12}\right)$ 
(e) No solution

5. Solve the following equation for x:

(a)  $x = \frac{5}{2} \pm \frac{i\sqrt{55}}{2}$ (b)  $x = \frac{4}{5} \pm \frac{i\sqrt{110}}{4}$ (c)  $x = \frac{5}{2} \pm \frac{i\sqrt{110}}{2}$ (d)  $x = \frac{5}{4} \pm \frac{i\sqrt{55}}{4}$ (e)  $x = \frac{5}{4} \pm \frac{i\sqrt{55}}{4}$ 



6. Find the equation of the parabola graphed below.

- 7. Which values of x satisfy the inequality  $x^2 6x + 5 \ge 0$ ?
  - $\begin{array}{ll} (a) & (-\infty,1] \cup [5,\infty) \\ (d) & [-\infty,1] \cup [5,\infty] \end{array} \\ \end{array} \\ \begin{array}{ll} (b) & (-\infty,1) \cup (5,\infty) \\ (e) & \text{None of the above.} \end{array} \\ \end{array} \\ \begin{array}{ll} (c) & (1,5) \\ (1,5) \end{array} \\ \end{array}$
- 8. Simplify  $\frac{x^2 + 15x + 50}{x + 5}$ . (a) 5x + 10 (b) 2x + 5 (c) x + 10 (d) 2(x + 5)(e) Cannot be simplified.
- 9. Simplify (4i 6)(12 + 2i). (a) -80 + 36i (b) 60i + 80 (c) 36i (d) 80i (e) 72 + 8i
- 10. (Tie Breaker Question #1) In how many distinct points does the line with equation

$$2023x + 2023y = 1$$

intersect the parabola with the equation

11. Solve for x.

(a) 
$$x = 6, 20$$
 (b)  $x = 6$  (c)  $x = 20$  (d)  $x = 3, 20$  (e)  $x = 3$ 

12. Express 
$$\frac{1}{25}$$
 in exponential form.  
(a)  $4 \times 10^{-2}$  (b)  $4 \times 10^{-1}$  (c)  $25 \times 10^{-2}$  (d)  $25 \times 10^{-1}$  (e)  $4 \times 10^{2}$ 

13. What is the center and radius of the circle  $(x-4)^2 + (y-3)^2 = 12?$ 

- (a) Center: (4,3), Radius:  $2\sqrt{3}$ (b) Center: (3, 4), Radius:  $2\sqrt{3}$
- (c) Center: (-3, -4), Radius: 12
- (e) Center: (4,3), Radius:  $\sqrt{3}$
- (d) Center: (3,4), Radius: 12

14. Solve for 
$$x$$
:  $\left|\frac{4x+3}{x+3}\right| = 3$ .  
(a)  $x = 3, -\frac{12}{7}$  (b)  $x = 3, \frac{7}{10}$  (c)  $x = 6, \frac{12}{7}$  (d)  $x = 6, -\frac{10}{7}$  (e)  $x = 6, -\frac{12}{7}$ 

15. Given 
$$f(x) = \frac{4x}{5-x}$$
. Find a formula for  $f^{-1}(x)$ .  
(a)  $\frac{5y}{4+y}$  (b)  $\frac{4y}{5-y}$  (c)  $\frac{5x}{4-x}$  (d)  $\frac{5x}{4+x}$  (e)  $\frac{5-x}{4x}$ 

16. If x is the average (arithmetic mean) of r and 1999, y is the average of 2r and 20, and z is the average of 3r and 3, what is the average of x, y, and z in terms of r.

(a) 
$$\frac{6r + 2022}{3}$$
 (b)  $\frac{r + 2022}{6}$  (c)  $6r + 2022$  (d)  $6r + 337$  (e)  $r + 337$ 

17. Suppose that x is an integer such that:

$$\frac{2x}{3} - \frac{x}{6}$$

(e) None of the above

is an integer. Which of the following statements is true about x?

- (a) x is even, but not necessarily a multiple of 3
- (b) x is a multiple of 12, but is not necessarily positive (c) x is negative
- (d) x is a multiple of 3, but not necessarily even
- 18. (Tie Breaker Question #2) Which of the following is closest to the decimal value of  $1.01^{100}$ ?
  - (a) 1 (b) 2 (c) **2.7** (d) 3 (e) 10

19. What is the sum of the exponents in the following expression once it has been completely simplified to have only positive exponents?

- 20. A rectangular garden has a perimeter of 34m. If the length were to be increased by 3m and the width were to be increased by 2m, the area would increase by 45  $m^2$ . Calculate the current length and width of the garden.
  - (a) width: 5m; length: 12m (b) width: 9m; length: 8m
  - (c) width: 8m; length: 9m (d) width: 2m; length: 15m
  - (e) Not enough information given
- 21. What is the product of the roots of the following polynomial?

(a) 
$$-6$$
 (b) 11 (c) 0 (d)  $-1$  (e)  $6$ 

 $r^3 - 6r^2 + 11r - 6$ 

22. What is the 2000<sup>th</sup> digit after the decimal point in the decimal representation of  $\frac{4}{7}$ ?

- (a) 5 (b) 1 (c) 4 (d) 7 (e) None of the above
- 23. Below are the first five terms (n = 1:5) in a sequence of numbers:

4, 3, 4, 7, 12.

If the sequence continues, which expression could be used to determine the nth term?

(a) 
$$n^2 - 4n + 7$$
 (b)  $|n-2| + 3$  (c)  $n^3 - 4n + 7$  (d)  $2n + 2$  (e)  $n - 1$ 

24. (Tie Breaker Question #3) Given that a and b are real numbers such that  $a^2 + b^2 = 2$ , determine the value of

$$\sqrt{a^4 + 8b^2} + \sqrt{b^4 + 8a^2}.$$

- (a) 2 (b) 4 (c) 6 (d) 8 (e) Not enough information is given.
- 25. When a certain 6-sided die is thrown, it can land on any one of its 6 sides with equal probability. What is the chance of the die landing on the same side three times in a row?
  - (a)  $\frac{1}{6}$  (b)  $\frac{1}{36}$  (c)  $\frac{1}{216}$  (d)  $\frac{1}{1296}$ (e) None of the above

(a

(e)

26. (Tie Breaker Question #4) What graph does the following equation create?

$$x^2 + 8x + 3y^2 - 6y + 7 = 0$$
) Ellipse (b) Circle (c) Parabola (d) Hyperbola ) None of the above

- 27. How many 3-digit positive integers do not contain the same digit twice in a row?
  - (a) 900 (b) 810 (c) 738 (d) 729 (e) 720
- 28. The Red River Delta is a strategically important area in the center of northern Vietnam that spans a length of roughly 120 km with a huge width of 140 km. A part of the river is used for tourism purposes. A boat service is installed to take tourists from wharf A to wharf B, which are 48 km apart. It takes a total of 5 hours for the boat to go from wharf A to wharf B, and back to A (assuming that the boat operates continuously). Find the speed of the boat in still water, given that water current has a speed of 4 km/h in the direction of wharf A from wharf B.

(a) 
$$20 \ km/h$$
 (b)  $30 \ km/h$  (c)  $40 \ km/h$  (d)  $50 \ km/h$  (e) None of the above

- 29. Given two points A and B, an ellipse is defined as the set of points P such that the sum of the distances from P to A and from P to B is a fixed number. Which of the following equations defines an ellipse?
  - (a)  $\sqrt{(x-3)^2 + (y-5)^2} = \sqrt{(x-2)^2 + y^2}$ (b)  $\sqrt{(x-3)^2 + (y-5)^2} + \sqrt{(x-2)^2 + y^2} = 9$ (c)  $\sqrt{(x-3) + (y-5)} + \sqrt{(x-2) + y} = 9$ (d)  $\sqrt{(x-3)^2 + (y-5)^2} - \sqrt{(x-2)^2 + y^2} = 9$ (e)  $(x-3)^2 + (y-5)^2 + (x-2)^2 + y^2 = 9$
- 30. (Tie Breaker Question #5) At a certain high school, senior prom was held in the school's gymnasium, which had an advanced automatic camera system installed. At the end of the party, each person was required to shake hands exactly once with every other person in the room. The camera system counted a total of 435 handshakes. How many students attended the prom that day?

(a) 
$$24$$
 (b)  $45$  (c)  $60$  (d)  $30$ 

(e) None of the above