

2023 Calculus Exam

1. Given $f(x) = (5x - 4)(3x^2 + 2)$ find $f'(x)$.

- (a) $15x^2 + 39x + 10$ (b) $45x^2 - 24x + 10$
(c) $24x^2 + 14x + 6$ (d) $30x$ (e) None of the above.

2. Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{(x+7)^2 - 49}{x}$$

- (a) 0 (b) 4 (c) 14 (d) 7
(e) Does not exist

3. Find the equation of the tangent line to the function $f(x) = 2x \sin(x)$ when $x = \frac{\pi}{2}$.

- (a) $y = 2x$ (b) $y = x$ (c) $y = 2x + \pi$ (d) $y = \frac{\pi}{2}x + \pi$ (e) $y = \frac{\pi}{2}x + 2$

4. Evaluate the limit.

$$\lim_{x \rightarrow 1} \frac{2x^3 + 8x - 10}{x^2 - 1}$$

- (a) 0 (b) 7 (c) $\frac{7}{2}$ (d) 2 (e) 6

5. Given $f(x) = (2x^3 + 7x^4 + 19x)^3$, find $f'(x)$.

- (a) $3(2x^3 + 7x^4 + 19x)^2$ (b) $3(2x^3 + 7x^4 + 19x)^2(28x^3 + 6x^2 + 19)$
(c) $\frac{1}{4}(2x^3 + 7x^4 + 19x)^4$ (d) $3(2x^3 + 7x^4 + 19x)^2(6x^2 + 28x^3 + 19x)$
(e) None of the above

6. What is the derivative with respect to x of the function $f(x)$?

$$f(x) = \frac{5x^3}{7x^2 + 2}$$

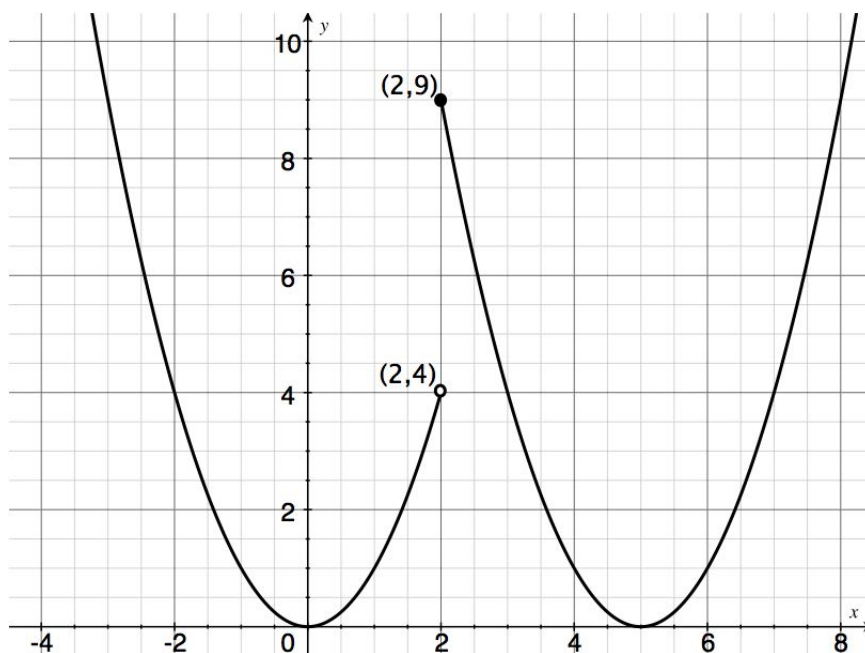
- (a) $\frac{15x^2}{14x}$ (b) $\frac{35x^4 + 30x^2}{(7x^2 + 2)^2}$ (c) $\frac{5x^3 - 7x^2 + 2}{7x^2 + 2}$ (d) $\frac{105x^4 + 70x^3 + 30x^2}{(7x^2 + 2)^2}$
(e) None of the above

7. (Tiebreaker #1) Evaluate the limit.

$$\lim_{x \rightarrow \infty} \left(\sqrt{x + \sqrt{x}} - \sqrt{x} \right)$$

- (a) 0 (b) $1/2$ (c) 1 (d) ∞
(e) Does not exist

8. What is the type and location of the discontinuity below?



- (a) removable discontinuity at $y = 4$ (b) removable discontinuity at $x = 2$
 (c) **jump discontinuity at $x = 2$** (d) jump discontinuity at $y = 9$
 (e) infinite discontinuity at $x = 2$

9. What values of a and b would make the function $f(x)$ continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & x < 2 \\ ax^2 - bx + 3 & 2 \leq x < 3 \\ 2x - a + b & x \geq 3 \end{cases}$$

- (a) $a = 1, b = 1$ (b) **$a = \frac{1}{2}, b = \frac{1}{2}$** (c) $a = 3, b = 3$ (d) $a = \frac{2}{3}, b = \frac{3}{2}$ (e) $a = \frac{3}{2}, b = \frac{2}{3}$

10. Find the derivative of $f(x)$ with respect to x .

$$f(x) = x - \frac{1}{3x^3} + 6x^4$$

- (a) $f'(x) = \frac{1}{x^4} + 24x^3$ (b) $f'(x) = 1 - \frac{1}{x^4} + 24x^3$
 (c) $f'(x) = 1 + \frac{1}{x^2} + 24x^3$ (d) **$f'(x) = 24x^3 + \frac{1}{x^4} + 1$**
 (e) $f'(x) = 1 - \frac{1}{x^4}$

11. Find the equation of the horizontal tangent line to the parabola $y = 2x^2 - 4x - 1$.

- (a) $x = 1$ (b) **$y = -3$** (c) $y = 1$ (d) $y = 4x - 4$ (e) $y = -1$

12. If $f(x) = \ln(2x)$ and $g(x) = e^x$, find the derivative of $f \circ g$ with respect to x .

- (a) 0 (b) **1** (c) $\frac{1}{2e^x}$ (d) $2e^{\ln(2x)}$ (e) $\frac{e^x}{2x}$

13. Find $f'(1)$ for $f(x) = \ln(-2 - x^4)$.

- (a) $\frac{-1}{3}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$ (e) **Undefined**

14. Differentiate the function $y = \csc(2x)$ with respect to x .

- (a) $-4 \cot(2x) \csc(2x)$ (b) $\cot(2x) \csc(2x)$ (c) $2 \cot(2x) \csc(2x)$ (d) **$-2 \cot(2x) \csc(2x)$**
(e) $\cos(2x) \sin(2x)$

15. Find the derivative of $x = \sec^3(e^{2t})$ with respect to t .

- (a) $6e^{2t} \sec^2(e^{2t})$ (b) $6e^{2t} \sec^2(e^{2t}) \tan(e^{2t})$
(c) $3e^{2t} \sec^3(e^{2t}) \tan(e^{2t})$ (d) **$6e^{2t} \sec^3(e^{2t}) \tan(e^{2t})$**
(e) $3e^{2t} \sec^2(e^{2t})$

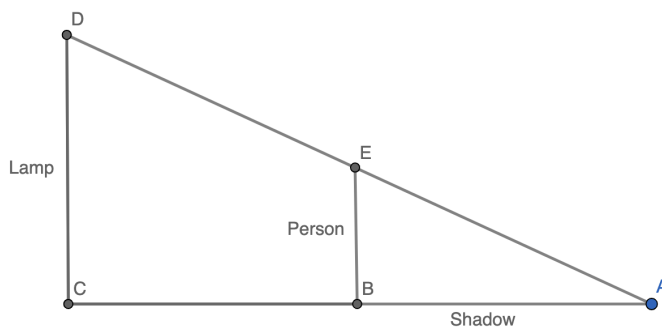
16. At what rate will the fluid level inside a vertical cylindrical tank drop if the fluid is pumped out at a rate of 3000 L/min? The tank has a radius of 5 meters. Note: 1000L = 1m³.

- (a) $\frac{25}{3\pi}$ m/min (b) $\frac{120}{\pi}$ m/min (c) $\frac{9}{25\pi}$ m/min (d) $\frac{25}{9\pi}$ m/min (e) **$\frac{3}{25\pi}$ m/min**

17. Estimate $\sqrt{1.25}$ by using the tangent line to $f(x) = \sqrt{x}$ at $x = 1$.

- (a) 1 (b) 1.05 (c) **1.125** (d) 1.2 (e) 1.25

18. **(Tiebreaker #2)** A 1.75 m person ($\overline{BE}$) is moving away from a 2.5 m tall lamp post ($\overline{CD}$) at a rate of 1.4 m/s. A light at the top of the lamp post (point D) casts a shadow in front of the person. Determine how fast the head of the shadow at point A is moving.



- (a) 3 m/s (b) 2.1 m/s (c) **$\frac{14}{3}$ m/s** (d) $\frac{4}{3}$ m/s
(e) Not enough information given

19. Evaluate the integral.

$$\int_1^5 \frac{5x}{2} dx$$

- (a) 25 (b) **30** (c) 0 (d) 100 (e) 120

20. Suppose $y = f(x)$ is graphed below:



Compute:

$$\int_0^4 f(x) dx$$

- (a) 3.5 (b) 1 (c) 1.5 (d) 2
(e) **None of the above**

21. Given $\int_{-1}^1 f(x) dx = 5$ and $\int_0^1 f(x) dx = 7$, calculate $\int_0^{-1} 3f(x) dx$.

- (a) 3 (b) **6** (c) -6 (d) -3
(e) None of the above

22. Evaluate and simplify:

$$\int_{-5}^5 \sqrt{25 - s^2} ds$$

- (a) π (b) **$\frac{25}{2}\pi$** (c) 5π (d) 5 (e) $\frac{\pi}{5}$

23. **(Tiebreaker #3)** Which of the following lines is tangent to the graph of $y = 2 + x^3$?

- (a) $y = \frac{1}{3}x$ (b) $y = \frac{1}{2}x$ (c) $y = x$ (d) $y = 2x$ (e) $y = 3x$

24. Which of the following is true of the function $f(x) = \ln(-2x)$?

- (a) $f(x)$ is decreasing (b) $f(x)$ is increasing
 (c) $f(x)$ is concave up (d) $f(x)$ has multiple inflection points
 (e) $f(x)$ has exactly 1 inflection point

25. On which of the following intervals is $f(x)$ concave up?

$$f(x) = 4 + 6x^2 + x^3$$

- (a) $(-\infty, \infty)$ (b) $(2, \infty)$ (c) $(-\infty, 2)$ (d) $(-\infty, -2)$
 (e) None of the above

26. Find all horizontal asymptotes of the function:

$$f(x) = \frac{1 - 2e^x}{1 - e^x}.$$

- (a) $y = 1$ and $y = 2$ (b) $x = 1$ and $x = 2$ (c) $x = 1$
 (d) $y = 2$ (e) There are no horizontal asymptotes.

27. Evaluate the limit.

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$$

- (a) 0 (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
 (e) The limit does not exist.

28. Find the derivative of y with respect to x , where

$$y = x^x.$$

- (a) $x^x(\ln(x) + 1)$ (b) $x \ln(x)$ (c) $x^x \ln(x)$ (d) x^x (e) $e^x \ln(x)$

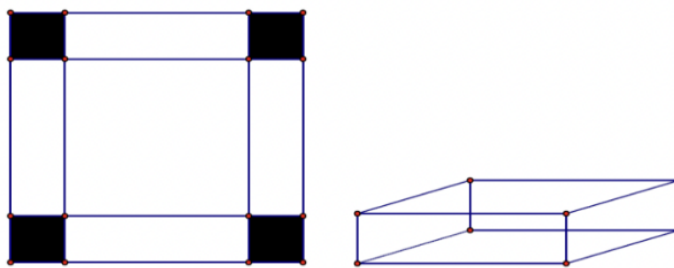
29. A baseball is hit by a bat and is accelerating upwards at the rate of $a(t) = 5t^3 + 22t + 15$ m/s² after t seconds. Which of the following formulas could describe the ball's velocity after t seconds?

- (a) $\frac{5t^4}{4} + 11t^2 + 15t$ (b) $15t^2 + 22$ (c) $\frac{t^3}{4} - 16t$ (d) $\frac{9}{2}t^2 + 11t$
 (e) None of the above

30. **(Tiebreaker #4)** A lifeguard is standing on the shore at a beach, and spots someone drowning 100 feet away down the beach, and 50 feet from the shore. The lifeguard plans to rescue them by running some distance along the beach and then swimming in a straight line to reach them. If the lifeguard runs at 13 ft/s and swims at 5 ft/s, how far should the lifeguard run along the beach in order to minimize the total time it takes to reach the drowning person?

- (a) 62 feet (b) $\frac{475}{6}$ feet (c) $\frac{150}{\sqrt{135}}$ feet (d) 75 feet (e) $\frac{125}{6}$ feet

31. Given a square aluminum plate with sides of 12 cm, a student cuts out the four corners. Each cut out is a square of the same size. The student then folds the aluminum plate, creating a box with no lid (as shown below). Find the maximum volume of the box.



- (a) 200 cm^3 (b) 128 cm^3 (c) 108 cm^3 (d) 144 cm^3 (e) 136 cm^3

32. Find the function $f(x)$ given:

$$f''(x) = 18x - 8, \quad f'(0) = 5, \quad f(0) = 1$$

- (a) $f(x) = 3x^3 - 4x^2 + 5x + 5$ (b) $f(x) = 5x^3 - 6x^2 + 7x + 2$
 (c) $f(x) = x^2 - 2x + 1$ (d) $f(x) = 3x^3 - 4x^2 + 5x + 1$
 (e) $f(x) = -x^2 + 2x - 5$

33. Evaluate the integral.

$$\int (4x - 2x^3)^3 (4 - 6x^2) dx$$

- (a) $3(4x - 2x^3)^2 + C$ (b) $\frac{(4x - 2x^3)^4}{4} + C$
 (c) $\frac{(4 - 6x^2)^4}{4} + C$ (d) $\frac{(4 - 6x^2)^4}{4}$ (e) None of the above

34. Let $f(x)$ be a function whose graph is a semicircle with endpoints $(a, 0)$ and $(b, 0)$ with $a < b$. Find:

$$\left| \int_a^b f(x) dx \right|.$$

- (a) $|f(b) - f(a)|$ (b) $(b - a)^2 \left(\frac{\pi}{2} \right)$ (c) $(b - a)^2 \left(\frac{\pi}{8} \right)$ (d) $(b - a) \left(\frac{\pi}{4} \right)$
 (e) None of the above

35. Evaluate the integral.

$$\int_{-\pi}^{\pi} \arctan(x) \, dx$$

- (a) 0 (b) 2π (c) 4π (d) -2π (e) Undefined

36. Evaluate the integral:

$$\int_0^1 \frac{e^t + 1}{e^t + t} dt$$

- (a) $\ln(e + 1)$ (b) $\ln(e + 1) - 1$ (c) e^1 (d) 1 (e) 2

37. Evaluate:

$$\frac{d}{dx} \int_1^{x^2} \ln(t) \, dt$$

- (a) $\ln(x^2)$ (b) $x^2 \ln(x)$ (c) $2x \ln(x^2)$ (d) $\ln(x^2)x^2 - x^2 + 1$
 (e) $2x \ln(x^2)x^2 - x^2$

38. Evaluate the integral.

$$\int_0^1 e^t \, dx$$

- (a) $e^t - e^0$ (b) $e - 1$ (c) e^t (d) $e^t + C$ (e) $e^x + C$

39. **(Tiebreaker #5)** Define S via the following integral:

$$S = \int_0^1 e^{x^2} \, dx.$$

Which of the following is true of S ?

- (a) $0 < S \leq 1$ (b) $1 < S < e - 1$ (c) $S = e - 1$ (d) $S > e - 1$
 (e) S is undefined

40. Find the derivative of the following function with respect to c : $f(c) = a^2c + bc^2$.

- (a) $2ac + 2bc$ (b) $a^2 + 2bc$ (c) $2ac + a^2 + 2c + 2bc$ (d) $\frac{-a^2}{2b}$
 (e) $2a + 2b$