

2024 Calculus Exam

1. What is the solution to the indefinite integral:

$$\int (4x^3 - 2x^4 + 3x^2 - x + 15) dx$$

- (a) $x^4 - \frac{2}{5}x^5 + x^3 - \frac{1}{2}x^2 + 15x + C$ (b) $x^4 - 2x^5 + x^3 - x^2 + 15x + C$
 (c) $x^3 - 3x^4 - 3x^3 - x^2 + 15x + C$ (d) $-x^4 + \frac{2}{5}x^5 + x^3 + \frac{1}{2}x^2 - 15x + C$
 (e) None of the Above

2. Solve the limit:

$$\lim_{x \rightarrow \frac{1}{2}} \frac{x^2 + 5x + 10}{x + 5}$$

- (a) $\frac{50}{22}$ (b) $\frac{51}{22}$ (c) $\frac{52}{22}$ (d) $\frac{53}{22}$ (e) $\frac{54}{22}$

3. What is the derivative of:

$$y = \sqrt{2x - \sin(x^2)}$$

- (a) $\sqrt{(2x - \sin(x^2))}(1 - x \cos(x^2))$ (b) $\sqrt{2x - \sin(x^2)}(1 - x \cos(x^2))$
 (c) $\frac{1 - x \cos(x^2)}{\sqrt{2x - \sin(x^2)}}$ (d) $\frac{1 - x \sin(x^2)}{\sqrt{2x - \sin(x^2)}}$
 (e) $\frac{2 - x^2 \cos(x^2)}{2\sqrt{2x - \sin(x^2)}}$

4. If a box with a square base with sides of length x and a height of $h = \frac{3}{2} - \frac{1}{4}y$, where $y = x - 2$, what is the maximum volume of the box?

- (a) $\frac{8}{3}$ (b) $\frac{64}{9}$ (c) $\frac{16}{3}$ (d) $\frac{64}{27}$ (e) $\frac{512}{27}$

5. What is the integral of the function $f(x) = x \sec(3x^2) \tan(3x^2)$?

- (a) $\frac{1}{6} \sec(x^3) + C$ (b) $\frac{1}{6} \sec(3x^2) + C$ (c) $\frac{1}{6} \ln(\csc(3x^2)) + C$
 (d) $\frac{1}{6} \ln(x \csc(3x^2)) + C$ (e) $\frac{1}{6} \ln(\csc(x^3)) + C$

6. What is the derivative of $y = \ln(\operatorname{arccot}(x^3))$?

- (a) $\frac{-3x^2}{(x^6 + 1) \arctan(x^3)}$ (b) $\frac{3x^2}{(x^6 + 1) \arctan(x^3)}$
 (c) $\frac{3x^2}{(x^6 + 1) \operatorname{arccot}(x^3)}$ (d) $\frac{-3x^2}{(x^6 + 1) \operatorname{arccot}(x^3)}$
 (e) $\frac{x^3}{(x^6 + 1) \arctan(x^3)}$

7. What is the derivative of $y = \frac{4x^4 - 2x^3 + 3}{x^3}$?

- (a) $\frac{16x^3 - 6x^2}{3x^2}$ (b) $4 + \frac{9}{x^4}$ (c) $\frac{16x^3 + 6x^2}{3x^2}$ (d) $\frac{8x^3 - 4x^2 + 3}{2x^3}$ (e) $4 - \frac{9}{x^4}$

8. For what value x is $y = 4x + 6$ tangent to $f(x) = \frac{x^3 + 2}{3}$?

- (a) 2 (b) 4 (c) -2 (d) -4 (e) 3

9. Locate the absolute extrema of the function $f(x) = 4x^3 + 9x^2 + 6x - 2$ on the closed interval $[-1, 0]$.

- (a) Max:(0, -2) ; Min:(-1, -3) (b) Max:(0, -2) ; Min:(-0.5, -3.25)
 (c) Max:(1, 3) ; Min:(0, 2) (d) Max:(0, -2) ; Min:(-0.5, -2.25)
 (e) None of the above

10. Consider the numbers x and y whose sum is 4 and let $P = x^2y$. For $-2 \leq x \leq 8$, what is the minimum value of P ?

(a) -100 (b) -16 (c) -128 (d) 0 (e) -256

11. Solve the integral:

$$\int_1^3 (x^2 - 3) dx$$

(a) $\frac{10}{3}$ (b) $\frac{7}{3}$ (c) $\frac{2}{3}$ (d) $\frac{8}{3}$ (e) $\frac{5}{3}$

12. Determine the interval at which the function $g(x) = x\sqrt{x+2}$ is concave up.

(a) $(-2, 2)$ (b) $(2, \infty)$ (c) $(-2, \infty)$ (d) $(-\infty, 2)$ (e) $(-\infty, \infty)$

13. If $\int_2^6 h(x)dx = k^2$ and $\int_2^{10} h(x)dx = -6 + k$, what is the value of $\int_6^{10} (x + h(x))dx$?

(a) $k^2 + k + 26$ (b) $-k^2 + k + 58$ (c) $-k^2 + k + 26$ (d) $-k^2 + k - 6$ (e) $k^2 + k - 6$

14. Integrate the following:

$$\int x\sqrt{x^2+1}dx$$

(a) $\frac{2}{3}x^{\frac{3}{2}} + C$ (b) $x\sqrt{2x} + C$ (c) $\frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$
 (d) $\frac{1}{3}(x^2+1)^{\frac{3}{2}} + C$ (e) None of the above

15. What is the third derivative of $f(x) = \sin(5x)$?

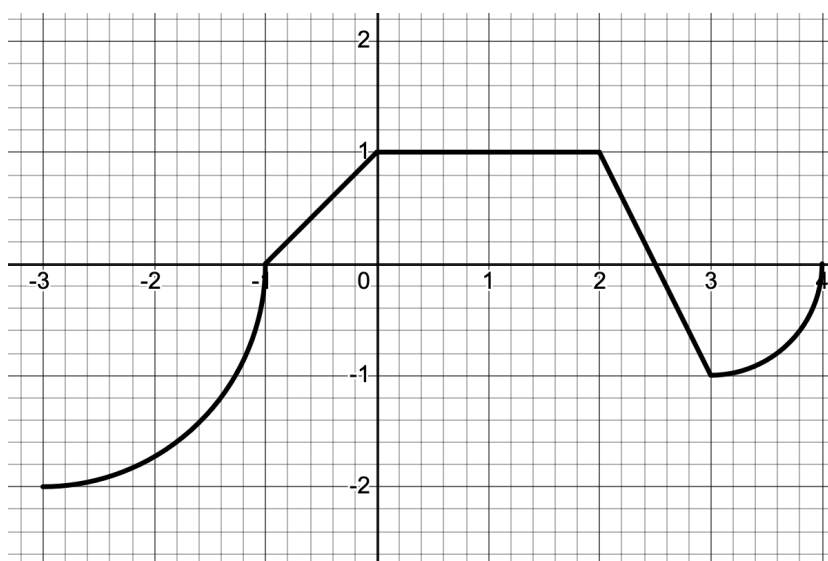
(a) $-125\cos(5x)$ (b) $-125\sin(5x)$ (c) $5\sin(5x)$ (d) $125\cos(5x)$ (e) $125\sin(5x)$

16. Which of the following is the value for y for the expression:

$$\int_2^y (4x + 2)dx = 28$$

(a) 6 (b) 5 (c) 8 (d) 3 (e) 4

17. Based on the graph of $f(x)$ below, what is the value of $\int_{-3}^4 f(x)dx$?



(a) $\frac{10+5\pi}{4}$ (b) $\frac{12+5\pi}{4}$ (c) $\frac{12-5\pi}{4}$ (d) $\frac{10-5\pi}{4}$ (e) $\frac{14+5\pi}{4}$

18. If $\lim_{x \rightarrow 0} \frac{e^{kx} - 1 + \sin(2x)}{x^2 + 2x} = \frac{7}{5}$, find the value of k .

- (a) 7 (b) $\frac{4}{5}$ (c) $\frac{14}{5}$ (d) $\frac{7}{5}$ (e) 2

19. Write an equation of the tangent line for the function $f(x) = 3 \tan x + 1$ at $x = \frac{3\pi}{4}$

- (a) $y - 4 = \frac{3}{2} \left(x - \frac{3\pi}{4} \right)$ (b) $y + 2 = 6 \left(x - \frac{3\pi}{4} \right)$
 (c) $y + 2 = 6 \left(x + \frac{3\pi}{4} \right)$ (d) $y - 4 = \frac{3}{2} \left(x + \frac{3\pi}{4} \right)$
 (e) $y + 4 = \frac{3}{2} \left(x - \frac{3\pi}{4} \right)$

20. Solve $\int \frac{1}{1+x^2} dx$.

- (a) $\arctan(x) + C$ (b) $\arccos(x) + C$ (c) $\arcsin(x) + C$ (d) $\cot(x) + C$ (e) $\csc(x) + C$

21. Find the limit:

$$\lim_{x \rightarrow -1} \frac{2x^3 + 8x^2 - 6}{5x^5 - 3x^3 + 2}$$

- (a) 0 (b) $-\frac{5}{8}$ (c) $\frac{11}{8}$ (d) 1 (e) $\frac{11}{17}$

22. If $f(x) = 3x^3 - 2x^2 + 5$, then what is $f''(2)$?

- (a) 26 (b) 32 (c) 23 (d) $\frac{32}{3}$ (e) $\frac{28}{3}$

23. Find $\frac{dy}{dx}$ using implicit differentiation:

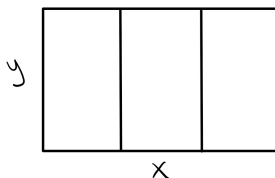
$$x^3 = \sin(x^3 y^3)$$

- (a) $\frac{1 - y^3 \cos(x^3 y^3)}{xy^2 \cos(x^3 y^3)}$ (b) $\frac{1}{y^3 \cos(x^3 y^3)}$ (c) $\frac{x(1 - y^3 \cos(x^3 y^3))}{y^2 \cos(x^3 y^3)}$
 (d) $\frac{1}{xy^3 \cos(x^3 y^3)}$ (e) $\frac{1}{y^3 \sin(3x^2 y^3)}$

24. A ladder is leaning against a wall. The bottom of the ladder is sliding away from the wall at a rate of 2 ft/s. If the ladder is 10 feet long and leaning against the wall at an angle of $\frac{\pi}{4}$ radians, how fast is the top of the ladder sliding down the wall?

- (a) $\sqrt{2}$ ft/s (b) 2 ft/s (c) $\frac{\sqrt{2}}{2}$ ft/s (d) $\frac{\sqrt{2}}{4}$ ft/s (e) 4 ft/s

25. A farmer with 800ft of fencing wants to enclose a rectangular area and divide it into three pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the three pens?



- (a) 40000 ft² (b) 10000 ft² (c) 30000 ft² (d) 15000 ft² (e) 20000 ft²

26. Consider the equation $x^2 + y^2 = 25$. Find $\frac{dy}{dx}$ at the point (3, 4).

- (a) $-\frac{4}{3}$ (b) $-\frac{3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$ (e) 0

27. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$.

- (a) **2** (b) 0 (c) $\frac{1}{2}$ (d) 1 (e) DNE

28. Calculate the area of the region between the curves $x = 2 + \frac{1}{4}y^2$ and $x = 4 - \frac{1}{4}y^2$.

- (a) $-\frac{16}{3}$ (b) 24 (c) **$\frac{16}{3}$** (d) -24 (e) 8

29. Evaluate:

$$\lim_{x \rightarrow \infty} 5x^{\frac{1}{\ln(x^2)}}$$

- (a) $\frac{1}{2}$ (b) 0 (c) **$e^{\frac{1}{2}}$** (d) ∞ (e) 1

30. Evaluate $\int \frac{3x}{\sqrt{4x^2 + 5}} dx$

- (a) $\frac{15}{\sqrt{4x^2 + 5}(4x^2 + 5)} + C$ (b) $\frac{1}{2} \sqrt[3]{(4x^2 + 5)} + C$
 (c) **$\frac{3}{4} \sqrt{4x^2 + 5} + C$** (d) $10\sqrt{4x^2 + 5} + C$
 (e) None of the above

31. The displacement of a particle moving along a straight line after t seconds, is given by the equation

$$s(t) = 5t^3 - 3t^2 + 8t$$

in meters. What is the velocity when acceleration is zero?

- (a) 12 m/s (b) 77 m/s (c) 43 m/s (d) 66 m/s (e) **7.4 m/s**

32. The velocity a particle is $v(t) = (t^2 - 4t + 4)$ m/s. What is the total distance traveled from $t = 0$ to $t = 2$ seconds?

- (a) $\frac{11}{12}$ m (b) **$\frac{8}{3}$ m** (c) $\frac{9}{4}$ m (d) $\frac{4}{3}$ m (e) 2 m

33. Compute this limit:

$$\lim_{x \rightarrow 3} \frac{x - 2}{x^2 - 5x + 6}$$

- (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ (e) **DNE**

34. Given the equation: $\frac{x^2}{16} + \frac{y^2}{9} = 1$. Find the area of the ellipse when $0 \leq x \leq 4$.

- (a) 8π (b) **6π** (c) 12π (d) 10π (e) 4π

35. Find the acceleration at $t = 5$ using the following displacement function.

$$d(t) = 4t^3 + 25t^2 + 5t + 2$$

- (a) 255 m/s^2 (b) 120 m/s^2 (c) **170 m/s^2** (d) 125 m/s^2
 (e) None of the above

36. Write the definite integral of the following Riemann Sum:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{8}{n} \right) \left[2 \left(\frac{8}{n} i \right)^3 + 9 \right]$$

- (a) $\int_0^2 (16x^3 + 9) dx$ (b) $\int_0^8 (16x^3 + 9) dx$
 (c) $\int_0^2 (8x^3 + 9) dx$ (d) $\int_0^2 (2x^3 + 9) dx$
 (e) **$\int_0^8 (2x^3 + 9) dx$**

37. Find the slope of the line tangent to the y -intercept of the parabola:

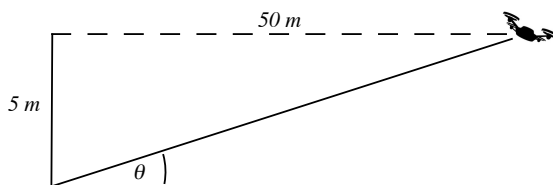
$$y = (x + 2)^2 - 4$$

- (a) 0 (b) 3 (c) 4 (d) 6
(e) None of the above

38. Find $f'(x)$ if $f(x) = 6 (\ln \sqrt{xe^x})$.

- (a) $f'(x) = 3 + \frac{3}{x}$ (b) $f'(x) = 3 \frac{x + e^x}{\sqrt{x}e^x}$ (c) $f'(x) = 3 \frac{xe^x + e^x}{2\sqrt{x}e^x}$
 (d) $f'(x) = 3 \frac{xe^x + e^x}{2xe^x}$ (e) None of the above

39. A drone flies directly overhead of a camera at a height of 5 meters. The drone continues in a straight horizontal line at a speed of 10 m/s. If the camera is tracking the drone, what is the rate of change of the angle θ the camera is making with the ground when the drone is 50 meters past the camera?



- (a) $-\frac{1}{\sqrt{55}}$ radians per second. (b) $-\frac{2}{\sqrt{55}}$ radians per second.
- (c) $-\frac{1}{5}$ radians per second. (d) $-\frac{2}{101}$ radians per second.
- (e) $-\frac{2}{\sqrt{2550}}$ radians per second.

40. For what value of k is the function continuous on the entire real line.

$$f(x) = \begin{cases} \frac{5 \sin(x)}{x} - 4, & x < 0 \\ k(x-1)^2 + 12, & x \geq 0 \end{cases}$$

- (a) -4 (b) 12 (c) -11 (d) 13 (e) -16