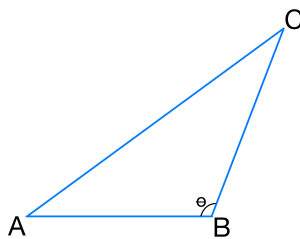


2023 Geometry Exam

- A playground has a roundabout (a circular piece of equipment that spins). Billy wants to find the area of the roundabout. He measures a distance of 4 feet from the center to the edge of the roundabout. Help Billy find its area. (Choose the closest answer.)
 - 12.5 ft^2
 - 16 ft^2
 - 48.5 ft^2 ¹
 - 23.6 ft^2
 - 64 ft^2
- Find the volume of a cylinder with a radius of 6 and height of 3.
 - 108π
 - 54π
 - 154π
 - 308π
 - 432π
- Find the area of a triangle with a base of 10 m and height of 7 m .
 - 70 m^2
 - 35 m^2
 - 25 m^2
 - 35 m^2 ²
 - $10\sqrt{6} \text{ m}^2$
- A circle has an area of $\frac{25\pi}{4}$, what is its circumference?
 - 2.5π
 - $\frac{5\pi}{4}$
 - 10π
 - 5π
 - None of the above
- If $AB = 10$ and $BC = 20$, what must the angle θ be for the area of $\triangle ABC$ to be 100?

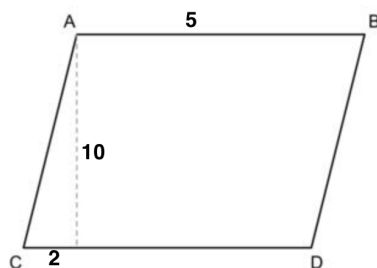


- 45°
 - 25°
 - 90°
 - 120°
 - No solution
- What is the length of the diagonal \overline{BD} for the square $ABCD$ with an area of 1?
 - $\sqrt{5}$
 - $\sqrt{2}$
 - 2
 - π
 - $2\sqrt{2}$
 - What is the area of the circle inscribed in a square of area 25?
 - 25π
 - 2.5π
 - 6.25π
 - 8.55π
 - 2.92π

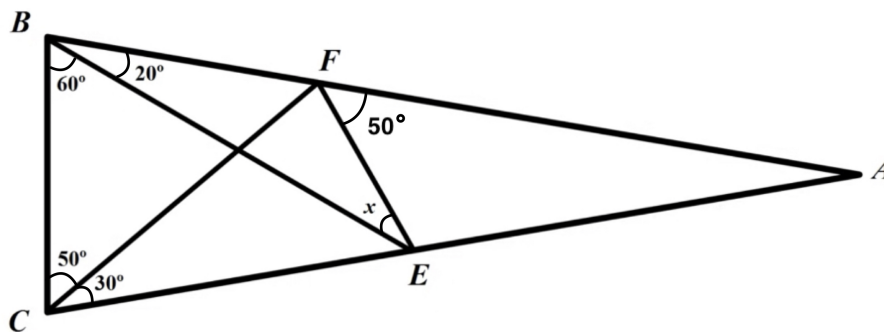
¹50 ft^2 would be a better answer.

²Oops. Both (b) and (d) were accepted.

8. Find the area of the parallelogram $ABDC$ (not drawn to scale).

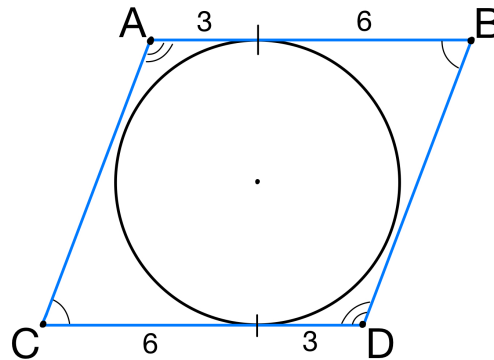


- (a) 70 (b) 50 (c) 75 (d) 100 (e) 60
9. A rectangle has a length of 20 units and a diagonal of length 25 units. What is the area of the rectangle?
- (a) 240 square units (b) 276 square units
(c) 300 square units (d) 375 square units
(e) 500 square units
10. In a right triangle one angle is 70° . What are the two remaining angles?
- (a) 20° and 90° (b) 83° and 27° (c) 15° and 75° (d) 55° and 55°
(e) Not enough information
11. A triangle has sides of lengths 5, 5, and 8. Find its area.
- (a) 10 (b) 12 (c) 12.5 (d) 20 (e) 40
12. (Tiebreaker #1) Find the measure x of $\angle BEF$ in the figure below.

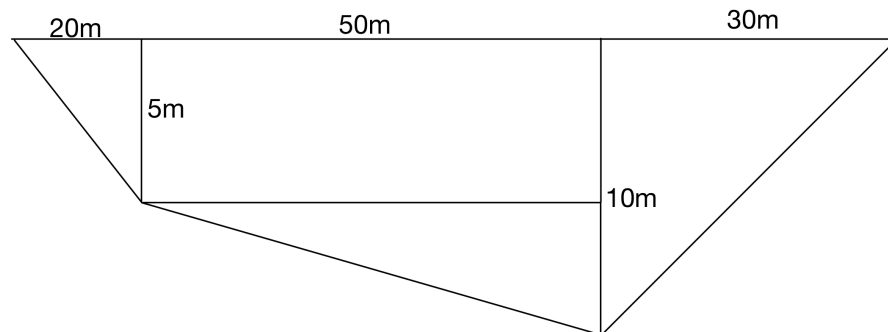


- (a) 20° (b) 25° (c) 30° (d) 40°
(e) None of the above

13. What is the area of the parallelogram $ABDC$ defined in the figure, given that the radius of the circle is 4?

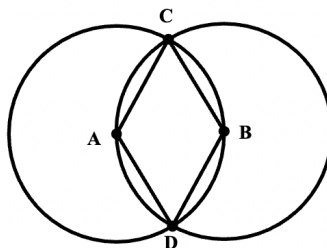


- (a) 51 (b) 34 (c) 81 (d) 72
 (e) Not enough information
14. In the parallelogram $ABDC$ above, what is the length of the diagonal \overline{AD} ?
- (a) 9 (b) 10 (c) 12 (d) $4\sqrt{13}$
 (e) Not enough information
15. A vertical cross-section of a stream was measured as shown below. Determine the cross-sectional area of the stream.

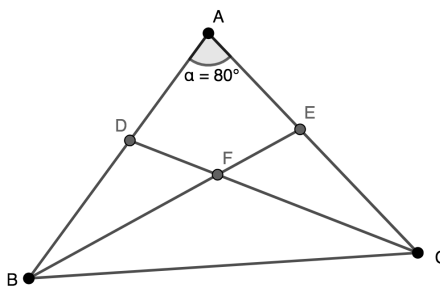


- (a) 900 m^2 (b) 575 m^2 (c) 500 m^2 (d) 450 m^2
 (e) Not enough information
16. Suppose that $ABCD$ is a trapezoid with $AB = BC = CD = 4$. Which of the following is not possible for the length of \overline{AD} ?
- (a) 1 (b) 4 (c) 6 (d) 8 (e) 14

17. The figure below shows two circles centered at points A and B . The measure of $\angle ACB$ is:



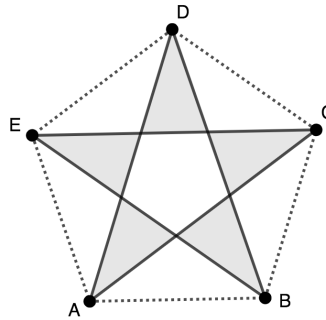
- (a) 60° (b) 45° (c) 30° (d) 50°
 (e) None of the above
18. The triangle $\triangle ABC$ is isosceles with $AB = AC$ and $m\angle CAB = 80^\circ$. If segments \overline{BE} and \overline{CD} are angle bisectors, intersecting at F , then what is the measurement of $\angle DFE$?



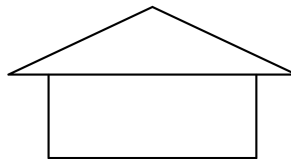
- (a) 100° (b) 110° (c) 120° (d) 130° (e) 140°
19. A garden is in the shape of a quadrilateral with two parallel sides of lengths 12 meters and 18 meters, which are 8 meters apart. If the garden is filled with soil to a depth of 20 centimeters, how many cubic meters of soil are needed?
- (a) 48 m^3 (b) 120 m^3 (c) 24 m^3 (d) 6 m^3
 (e) Not enough information
20. **(Tiebreaker #2)** A certain pizza parlor makes two sizes of circular pizzas. A Small pizza has a diameter of 12", and is cut into 6 equally sized slices. Meanwhile, a Large pizza has a diameter of 14", and is cut into 8 equally sized slices. What is the ratio of the area of a slice of a Small pizza to the area of a slice of a Large pizza?

- (a) $3 : 4$ (b) $6 : 7$ (c) $36 : 49$ (d) $48 : 49$ (e) $1 : 1$

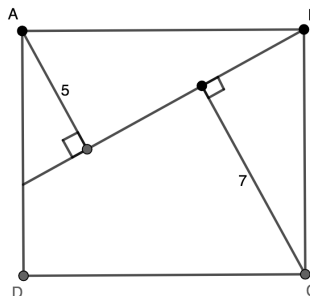
21. A 5-pointed star is formed by connecting the vertices of a regular pentagon $ABCDE$ as shown below. Find the measurement of $\angle CAD$.



- (a) 24° (b) 30° (c) 36° (d) 45°
 (e) Cannot be determined
22. What is the contrapositive of the statement:
 “If a triangle is isosceles, then it has two congruent sides.”
- (a) If a triangle does not have two congruent sides, then it is isosceles.
 (b) If a triangle is not isosceles, then it does not have two congruent sides.
 (c) If a triangle does not have two congruent sides, then it is not isosceles.
 (d) If a triangle is isosceles, then it has two congruent sides.
 (e) If a triangle has two congruent sides, then it is isosceles.
23. A drawing of a house consists of an isosceles triangle stacked directly on top of a rectangle (see figure). The rectangle has length 20 cm and height 10 cm . The roof is a triangle with a base of 24 cm and height of 5 cm . Find the perimeter of this house in cm .

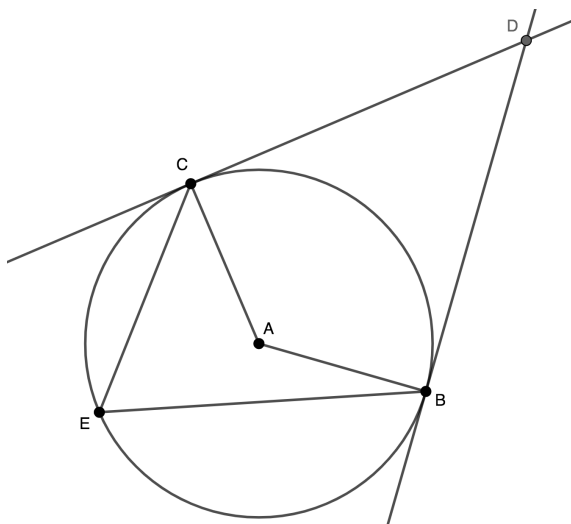


- (a) 70 (b) 76 (c) 69 (d) 90 (e) $44 + 10\sqrt{5}$
24. **(Tiebreaker #3)** Find the area of square $ABCD$ shown below. (Measurements shown are in cm .)

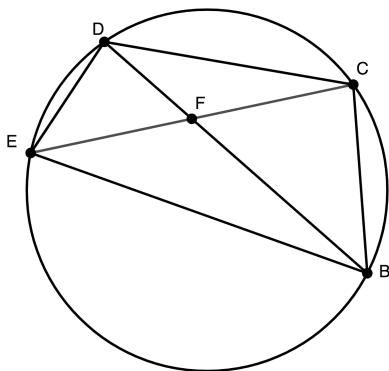


- (a) 32 cm^2 (b) 74 cm^2 (c) 35 cm^2 (d) 85 cm^2 (e) 72 cm^2

25. In the figure below, lines \overleftrightarrow{BD} and \overleftrightarrow{CD} are tangent to the circle centered at A , at the points B and C respectively. Given that the measure of $\angle BAC$ is twice the measure of $\angle BDC$, find the measure of $\angle BEC$.

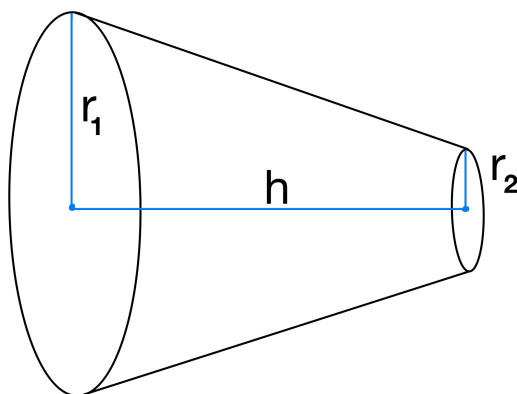


- (a) 60° (b) 64° (c) 65° (d) 68° (e) 70°
26. The quadrilateral $BCDE$ is inscribed in a circle, and its diagonals intersect in the point F . Which pairs of triangles must be similar?



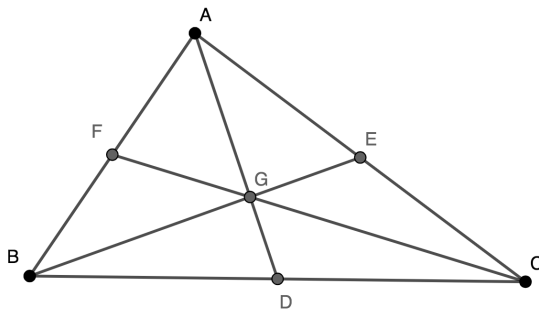
- (a) $\triangle BDE \sim \triangle BCE$ (b) $\triangle CFD \sim \triangle BFE$
 (c) $\triangle BCF \sim \triangle BEF$ (d) $\triangle DEF \sim \triangle CBE$
 (e) $\triangle CBD \sim \triangle EBD$
27. In triangle ABC , $m\angle B = 60^\circ$, $m\angle C = 30^\circ$, and side $AB = 10$ cm. Find side AC .
- (a) $5\sqrt{3}$ cm (b) 10 cm (c) $10\sqrt{2}$ cm (d) $10\sqrt{3}$ cm (e) 20 cm

28. Suppose that square $ABCD$ has an area-to-perimeter ratio of $2 : 1$. What is the area-to-perimeter ratio of the circle inscribed inside square $ABCD$?
- (a) $\pi : 1$ (b) $\pi : 2$ (c) **$2 : 1$** (d) $1 : 1$ (e) $4 : 1$
29. Which of the following terms refers to a line segment that connects two non-adjacent vertices of a polygon?
- (a) Median (b) **Diagonal** (c) Apothem (d) Perpendicular Bisector
(e) Altitude
30. A new highway development plan intends to connect two existing parallel highways that run East to West and which are separated by 20 miles. The proposed new route will run 60 degrees East of true North connecting the two highways. What is the length of the new stretch of highway in miles (rounded to the nearest whole number)?
- (a) 20 (b) 28 (c) 30 (d) 35 (e) **40**
31. Determine the volume of the nozzle shown below if $h = 3 \text{ cm}$, $r_1 = 2 \text{ cm}$ and $r_2 = 1 \text{ cm}$. (The nozzle is a section of a right circular cone.)



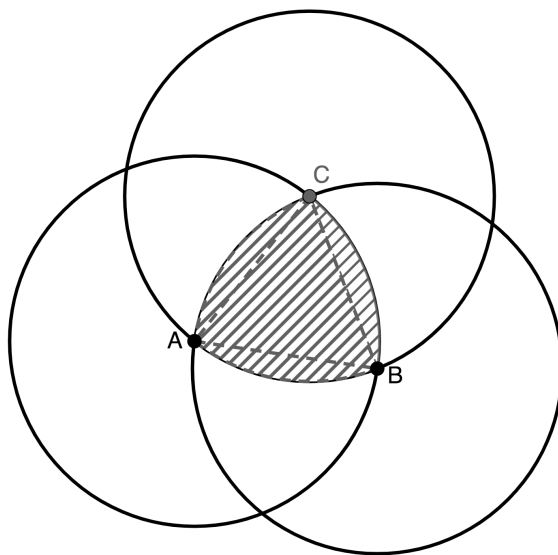
- (a) $4\pi \text{ cm}^3$ (b) $6\pi \text{ cm}^3$ (c) **$7\pi \text{ cm}^3$** (d) $8\pi \text{ cm}^3$
(e) Not enough information
32. **(Tiebreaker #4)** Refer to the picture of a nozzle in the previous problem. If the output radius is 5 cm and the input radius is 3 cm , what is the required length h of the nozzle so that its volume is $98\pi/3 \text{ cm}^3$?
- (a) 4 cm (b) **2 cm** (c) 1 cm (d) 5 cm
(e) Not enough information

33. Suppose $\triangle ABC$ has sides $AB = 8$, $BC = 12$ and $CA = 10$. When we draw the three medians \overline{AD} , \overline{BE} , and \overline{CF} of $\triangle ABC$, it divides the triangle into 6 non-overlapping smaller triangles.



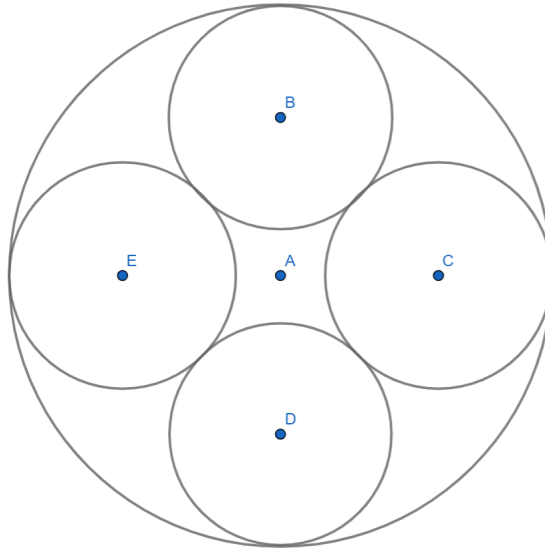
Which of these 6 triangles has the largest area?

- (a) $\triangle BDG$ (b) $\triangle CEG$ (c) $\triangle AFG$ (d) $\triangle AFG$ and $\triangle BFG$
 (e) They all have the same area.
34. Three circles of radius 1 are shown below, centered at points A, B and C . What is the area of the shaded region?



- (a) $\frac{\pi - \sqrt{3}}{2}$ (b) 1 (c) $\pi/6$ (d) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (e) $\sqrt{3}/4$

35. **(Tiebreaker #5)** Refer to the following figure.



Let the circles centered at points B, C, D , and E have radius r , and let them be tangent to one another and the circle centered at A . What is the ratio between the area of the circle centered at A and the area of the region bounded between the circles centered at B, C, D , and E which contains the point A ?

- (a) $\frac{(3 + 2\sqrt{2})}{4}$ (b) $\frac{\pi(3 + 2\sqrt{2})}{2 - \pi}$ (c) $\frac{\pi(3 + 2\sqrt{2})}{4 - \pi}$ (d) $\frac{\pi(3 - 2\sqrt{2})}{4 - \pi}$
 (e) None of the above